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Waves on water of varying depth: a report on an IUTAM symposium

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This is a review of an IUTAM symposium held in Canberra on 20–23 July 1976. The subject matter ranged widely: from surf on beaches to waves of oceanic scale, and from elegant analysis to observations from tide gauges.

1. Introduction

It is not an accident that over many years Australian theoretical research and observational research have contributed significantly to the topic of this symposium. They have been stimulated by her 20000 km long shoreline facing three of the earth's four oceans, in particular by her unique coast along the Southern Ocean and her substantial continental shelf. The meeting was conceived in the early Seventies and eventually found strong financial support from IUTAM as well as locally from Esso Australia Ltd, the Broken Hill South Ltd, the Shell Development (Australia) Pty Ltd, the Broken Hill Pty Co. Ltd and the Australian Academy of Science, which provided an excellent venue and supporting facilities. About 50 people, active in the field of water waves, attended this symposium, the first IUTAM symposium in the Southern Hemisphere. The dozen or so visitors from abroad enjoyed a friendly welcome and hospitality. Lectures were delivered in the Australian Academy of Sciences, which together with the dining and residential facilities of the nearby University House of the Australian National University provided nearly ideal conditions for the participants.

The topic for the symposium can include a great variety of subjects, and this was reflected in the material presented by the speakers. The papers covered wave motion on a wide range of scales, from small laboratory experiments and ordinary surface gravity waves to tides and topographic planetary waves of very long period. Some papers included no depth variation or even no finite depth, but despite this diversity there was much common ground. All examples are relevant to waves approaching or travelling along coasts and further coherence is provided by the mathematics used to describe the waves although this varies from the simplest possible equations through involved nonlinear approximations to elegant exact solutions.

Among the problems discussed, two which received much attention were the generation and propagation of tsunamis and of long waves on the continental shelf. The latter are of special interest in Australia for a number of reasons. Quite a lot of the pioneering work on this subject was done in Australia, and the Australian coastline provides some excellent examples in the form of observations taken from tidal records. The mechanism of generation of these waves is not fully understood and new phenomena are being discovered as the available data are further analysed.

There is still a lot to be discovered about the basic physical processes occurring in waves on water of variable depth, and the common thread running through the symposium was the search for an understanding of these fundamental processes. Time and again speakers were heard to say that further experiments were needed to check this or that result, or that a new theory was needed to explain some experimental results. These sorts of comments included both field observations, with all their inherent complications, and laboratory experiments made under carefully controlled conditions. Our understanding is still limited, but the papers of the symposium represent some of the latest work in a multi-pronged attack on the problems.

Our mathematical techniques also leave much to be desired. In all cases approximations need to be made and in an appreciable number of the talks the simplest possible equations, the linear long-wave equations, were used. The discussion of theoretical and experimental work for simple three-dimensional geometries, such as islands, showed the difficulties inherent in the subject. Although numerical solutions of these equations can be obtained for most specific problems, there is a clear need for analytical methods to be developed to improve our understanding. Even where analytical results can be obtained, the approximations necessary leave some doubt as to their utility in particular examples, so that careful experiments are desirable.

In the following report of the symposium, papers will be discussed in roughly chronological order, but with some rearrangement under various headings. Most papers were invited, but an opportunity was given for other participants to present work and these papers have also been included. A full list of the papers presented is given in the appendix and these papers are indicated by an asterisk in the text. The full proceedings will be published by the Australian Academy of Science, Canberra.

2. Gravity wave propagation in water of variable depth

Appropriately the first speaker, E. O. Tuck* (University of Adelaide), is an Australian. He outlined the rather limited range of exact solutions to the linearized surface wave problem for water of variable depth. The linear long-wave (or shallow-water) equations are more amenable to analysis, but at first sight are limited in that only vertical walls or slowly varying depths may be included. However, regions where the depth varies appreciably over one wavelength can be handled by matching the shallow-water 'outer' solution to a depth-dependent potential 'inner' solution, valid in the rapidly varying region. Details of matching such solutions were given for some examples. In particular, by considering a certain limiting case, Tuck showed that the appropriate shallow-water matching conditions across a discontinuity in depth are that the surface elevation and mass flux be continuous (as in Lamb 1932, p. 262).

Applications of some of these matching ideas to ship motions were also reviewed. In particular, it was noted that an Australian mathematician, Michell (1898), had looked at such problems and solved certain airfoil problems that arise in this context *before* the corresponding aerodynamic theory was developed. The discussion centred around the question as to why Lamb's step condition circumvented the need for studying the inner problem, and there was speculation as to the corresponding step condition when the shallow-water constraint was relaxed.

For linear non-dispersive water waves in branching channels, there is a complete mathematical analogy with electromagnetic circuitry. Thus fluid mechanicians can use to advantage many of the concepts and techniques developed in the context of electromagnetism. In his talk R. W. Preisendorfer* (University of Hawaii) presented a numerical method using the technique known as 'invariant imbedding'. Its operation was carefully explained for finding periodic solutions of one-dimensional long-wave equations. In essence it relies upon finding a scattering matrix at each point. Indications were given of how it might be generalized to solve some two-dimensional equations (Preisendorfer 1975).

Extending his own work (Christiansen 1975) to somewhat more complicated topographies, P. L. Christiansen* (Technical University of Denmark) investigated the combined refraction and diffraction of long gravity waves in water over certain 'power-law' circular-symmetric depth profiles. Results were obtained for islands with a discontinuous slope and also for discontinuities in the depth. Using the geometrical theory of diffraction (Keller 1962) a solution was obtained which included creeping waves around the discontinuities, and the decay exponent and diffraction coefficient for these creeping waves were found. In general, the rays from a point source were shown to be sine-spirals, while the problem of planar incident waves was included by letting the source point tend to infinity. The solutions presented were for circular-symmetric topographies having zero depth at the origin. This restriction arises from the power-law dependence of depth on radial distance. The case of finite depth at the origin was mentioned, and the great technical difficulties pointed out. Using so-called 'canonical' quantities the solutions for more general topographies may be found, but no details were given. The method is an asymptotic short-wavelength theory, and the basic equations are those of linearized long-wave theory. Great care would be needed, therefore, in the choice of length scales for any practical application. It was pointed out in discussion that the effects of creeping waves would be very small compared with the refracted wave field in any practical problem.

Some experiments designed to find resonances predicted by a geometric theory for water waves refracted around an island (Shen, Meyer & Keller 1968) were presented in the informal session by D. G. Provis* (Flinders University). The theory failed to describe the observed results and it was suggested that diffraction effects were significant. Much better agreement was obtained by using solutions of the long-wave equations in the limiting case of an island of zero radius. It was demonstrated that Battjes (1968) gives an approach which provides a guide to the applicability of geometric approximations.

3. Tsunami generation and propagation

When water waves are discussed anywhere in the vicinity of the Pacific Ocean, there is a good chance that tsunamis, that is long gravity waves due to undersea earthquakes, will be mentioned. The greatest damage caused by tsunamis has been in Japan, so that it is no surprise that the Japanese contributions were on this topic.

A wide-ranging survey of research and observations was presented by Y. Nagata* (Tokyo University). Statistical analysis provides rough relationships between earthquake magnitude and focal depth and resulting tsunami magnitudes (Iida 1961). Certain earthquakes then stand out as being more efficient at tsunami production than is usual. More detailed study has shown that the time scale of the ground motion may be the most important factor neglected (Kanamori 1972). Ground motions with time scales between 10 and 100 s are most effective. Other important effects are due to the distribution of ground motion and refraction by variations of water depth, which both lead to directional variation of the wave generated (Aida 1969).

The development of seismology is reaching the point where adequate estimates of the ground motion may be made directly from observations of seismic waves in sufficient time to predict the resulting tsunamis for reasonably distant areas. This gives added interest to the work of Alexeev & Gusiakov* (USSR Academy of Sciences, Novosibirsk) on the numerical simulation of tsunami generation. One model described was for a simple uniform topography but included the motion in the earth and the compressibility in the water. A much richer spectrum of waves occurs in this case. The solution for particular earthquake mechanisms was obtained by multiple Fourier transforms with numerical evaluation of the integrals. An indication of the level of international cooperation in this field was given by the final example: a computation for a tsunami in the region of the South Kurile Islands using a program due to Bernard (1975) at Hawaii.

When a tsunami arrives, or is created, close to the coast, nonlinear theory is required in order to predict adequately the region of inundation. By choosing dimensions appropriate to large earthquakes off the Japanese coast, K. Kajiura* (Tokyo University) showed that the linear long-wave equations are sufficient except close to the shore, where the finite-amplitude shallow-water equations need to be used. A variation of Carrier & Greenspan's (1958) transformation for the latter gives equations which are easier to handle. In particular, if it is assumed that there is no reflexion, a number of explicit results for the run-up may be found. If the bottom slope is no less than 0·02, which is the case for much of Japan, then there is no need to consider bore formation or wave breaking since the theory can be uniformly valid.

The theme of tsunami propagation in three dimensions was taken up by A. T. Chwang & T. Wu* (California Institute of Technology). Refraction can produce converging wave fronts: the simplest example is a cylindrically symmetric wave. Preliminary calculations with the nonlinear Boussinesq wave equations and results of experiments were described. The initial circular wave had a solitary-wave profile. It propagated to the centre and was reflected. In a constructive discussion it was suggested that the interpretation of results would be assisted by comparisons with the corresponding linear solution and with the two-dimensional interaction of solitary waves travelling in opposite directions.

4. Waves on beaches

A number of speakers discussed problems relevant to waves on beaches, an area of considerable economic and social importance. The widest field was covered by A. J. Bowen* (Dalhousie University), who discussed various types of resonant wave interactions in the inshore zone. These interactions are thought to play an important role in the formation of coastal features such as crescentic and longshore bars, and beach cusps. The interactions involve edge waves, which may propagate in either direction along the shore, incident waves, reflected waves and the interaction of these waves with currents and topography. A number of the possible interactions have been studied theoretically and by experiment, both in the laboratory and using field measurements. In experimental observations, the most studied interaction is the subharmonic generation of edge waves of frequency $\frac{1}{2}\sigma$ by an incident wave of frequency σ. The interactions are nonlinear, but occur at second order rather than third order as is the case for deep-water waves. Experiments were reported where the maximum amplitude of the edge waves was given as a function of incoming wave amplitude. It was shown that the edge-wave amplitude reaches a limiting value and then decreases when the incident waves begin to break. The surf zone appears to give an effective viscosity in the inshore region and the width of the surf zone is thus an important parameter. The preferred modes are edge waves having offshore scales large in comparison with the surf zone.

Wave breaking is one prominent wave phenomenon which we still do not fully understand. In an introduction to the talk by J. D. Fenton, M. S. Longuet-Higgins (Cambridge University) briefly presented a numerical method developed to calculate irrotational free-surface flow, with no limitation on free-surface slopes (Longuet-Higgins & Cokelet 1976). The results shown included a deep-water wave to which excess energy had been given by a suitable surface pressure distribution. The calculation had been terminated after the crest of the wave had developed an overhanging jet of water, when the curvature of the jet's tip had become too greato fr resolution by the numerical scheme. A film shown in the informal session (Longuet-Higgins*) confirmed a result of the computation: that an overhanging water surface is possible without any singularity in that surface.

Fenton proceeded to describe how he and Mills* (Monash University) were extending the method to include finite depth. Differences in detail were given. An ambitious calculation describing a solitary wave propagating over a horizontal bottom terminated by a steep slope had been started, but numerical instability was causing problems.

Most of Longuet-Higgins'* informal presentation concerned the mean force due to waves on a floating or submerged body. The direction and magnitude of the mean force depends on the proportion of the incident waves' energy that is reflected, transmitted, absorbed or dissipated by wave breaking. These effects were demonstrated in a short film. Particularly striking examples were a submerged cylinder experiencing a force in the direction opposite to the waves' direction of propagation, and a 'sand bar' on wheels which moved steadily shorewards under the influence of long waves but seawards under the influence of short breaking waves.

An interesting and novel approach to the problem of wave breaking was provided by H. G. Hornung & P. Killen* (Australian National University). A wave which is obliquely incident on a sloping beach will break first at one end of the beach; the point of breaking then moves along parallel to the bottom contours. Relative to the point of breaking, the water has a velocity towards the wave which is the wave speed divided by the cosine of the angle between the wave crest and the bottom contours. Using this fact, an oblique stationary breaking wave was formed by placing an approximately wedge-shaped obstacle at an angle in a high Froude number stream in a laboratory flume. The exact shape of the obstacle was determined by trial and error, since no theoretical basis exists from which to design such a feature. Indeed, the form of the breaking wave itself remains largely undetermined in the field. Personal experience and comparison with photographs were used to define the required wave form. Having obtained a satisfactory stationary breaking wave in the flume, detailed measurements were made of both the shape of the surface and the streamlines of the flow. The contours of the surface are not unlike the characteristics near shock waves in gasdynamics and certain other analogies were noted. A dividing stream surface was found which determined that part of the flow which went into the turbulent curl and that which did not. Model surfboards were constructed and by suitable placement of scaled weights, the models could be made to ride the breaking wave completely unsupported. By careful measurement of the position of the board, the forces acting on it could be determined without the need for direct measurement.

D. H. Peregrine reported progress in a continuing project with S. Hibberd* (Bristol University). The run-up on a beach after a wave has broken is modelled by a bore running up the beach under gravity. The bore is described using the finite-amplitude shallow-water equations. Numerical computations of the run-up of the bore can be compared with the available analytical results (Meyer & Taylor 1972). The method used for the computation is a Lax–Wendroff scheme with mass and momentum conserved. A dam-break solution is used at the point of impact of the bore on the still-water shoreline. When the run-up tip gets very thin it is truncated, but no significant errors have been detected from this procedure. During the backwash, the computations show the formation of a backwash bore. A preliminary computation with a series of periodic bores has produced a realistic pattern for the run-up. One very interesting feature of that result is that the maximum water velocity occurs during the backwash, and this may be of significance in sediment-transport problems.

5. Waves and currents

'Geometrical' or 'ray' methods came to the fore in the papers on the interaction of waves and currents; allowance for depth variation leads to even greater complexity. However, I. Jonsson* (Technical University of Denmark) sees the practical benefits of making wave-refraction calculations as accurate as possible. He gave a rapid review of the substantial amount of work done at the Technical University of Denmark on making such calculations more complete. Particular note was made of the care needed to use a horizontal datum level and of the difference between ray directions and wave orthogonals. Substantial details were then given for two-dimensional examples (i.e. only one horizontal dimension) of periodic waves on steady currents with a non-uniform bed. Solutions have been found for flow with uniform vorticity. Although the algebra is daunting, the final results are reassuringly similar to those for irrotational flow. Indeed the expression for the wave-action density of infinitesimal waves has exactly the same form, viz. (energy density)/(wave frequency relative to the water), as long as the average velocity of the water is used to define the frequency.

This paper stimulated discussion ranging from the underlying assumptions of the work to B. V. Hamon's practical concern with the effect of wave set-down on tide gauges situated in harbours.

When depth is allowed to drop out of consideration, by taking waves on deep-water currents, the problems of refraction are considerably simplified. This is especially so for slowly varying finite-amplitude waves since the periodic wave solution depends on one dimensionless parameter instead of two, and wave-induced currents need not be considered. D. H. Peregrine (Peregrine & Thomas,* Bristol University) applied Whitham's averaged-Lagrangian method for slowly varying finite-amplitude waves and particularly highlighted the solutions for waves near caustics.

In general, the solutions are not valid there unless higher-order dispersion (or diffraction) terms are included. Analysis of the near-linear (or weakly non-linear) case distinguished two general types of caustic. One type gives a singular solution with finite wave steepness. The second type is more disturbing in that there is no singularity in the solution, even though the linear solution is singular;

higher-order infinitesimal solutions show reflexion. These may occur for *all* simple types of nearly linear waves.

Solutions were presented for each of these two types, using the finite-amplitude numerical solutions of Longuet-Higgins (1975). These cover the whole range of periodic travelling waves, and it is also reasonable to identify the wave of maximum energy with the limit of non-breaking waves. The solution for the first type of caustic, one formed by the current shear $V(x)\mathbf{j}$, agrees well with nearly linear analysis. The example of the second type, a 'stopping' velocity on a current $U(x)\mathbf{i}$, is more complicated. Perfectly plausible solutions, which show waves increasing slowly until they break, are part of a continuous range of solutions which also include spurious solutions growing near the linear caustic. The latter are also discussed in a recent paper by Smith (1976). The reflected waves, when they exist in this case, show even more bizarre behaviour, a singularity existing at finite amplitude, with no hint as to what happens physically. Further reflexion is not possible.

The lively discussion re-emphasized short-comings of the method used, particularly the need to include higher-order dispersion terms and the likely importance of including possible wave asymmetry in the wave solution to be averaged.

Complications beyond a simple caustic were revealed by R. Smith* (Cambridge University). Waves oblique to a current $U(x)\mathbf{i}$ of finite depth can meet a situation equivalent to the merging of two caustics, essentially similar to a caustic cusp. Parameterizing and generalizing ray solutions by integrating leads to a uniformly valid representation for linear waves equivalent to an ansatz involving Pearcey's (1946) cusp function. Detailed numerical results were given for the region with strong amplification. For deep-water waves the results are identical with the Fourier-transform calculations of Hughes (1976).

Substantial discussion centred on one point to which Smith had drawn attention in his talk. If ϵ is an appropriate small parameter describing the shortness of the wave relative to the current (or topographic) length scale, then a typical ray solution is obtained with the various wave properties expressible in a series of powers of ϵ^2. For an ordinary caustic the corresponding parameter is ϵ and for cusped caustics $\epsilon^{\frac{1}{2}}$. There is thus a difference of *four* orders of magnitude between an error term in a simple ray solution and the error term for cusp problems. This is of particular importance in experimental and practical cases, where ϵ is often not very small. The difference of a single order of magnitude for caustics may help to explain the difficulties Provis* had in comparing theory and experiment.

6. Waves in a rotating stratified medium

Not content with the complexities of waves on a free surface over a variable bottom, three speakers introduced the further complication of density stratification. The papers covered the whole range from elegant analysis, through heavy computation, to direct attempts to understand observed waves.

M. Roseau* (Université de Paris) presented recent theoretical results on the propagation of waves of subinertial frequency (i.e. $\omega < f =$ Coriolis parameter) in a two-layer, uniformly rotating fluid exterior to a wedge formed by two vertical planes. Since the fluid considered was of constant depth, Roseau was able to separate the problem into two boundary-value problems, one for the vertical and one for the horizontal dependence. The vertical problem gives the usual two modes (one barotropic and one baroclinic) for a two-layer fluid. However, the second problem, involving the boundary conditions of zero normal velocity on the wedge and boundedness at infinity, is far from trivial. Following a procedure developed earlier by Roseau (1967), the solution was represented in the form of two Laplace-type contour integrals in which the unknown paths of integration and analytic functions in the integrands were subsequently determined by the boundary conditions. From asymptotic expansions of the integral representation Roseau showed that, far from the corner, the solution, for either vertical mode, consisted of waves of edge-wave type, travelling parallel to the walls of the wedge and decaying exponentially away from the walls. For a straight wall the solution appeared to reduce to that for surface and internal Kelvin waves, although this identification was not made explicit by the author. In the discussion it was pointed out that Packham & Williams (1968) have also given a solution to the second problem.

Even within the confines of linearized theory, wave motion on continental shelves presents substantial analytical difficulties for moderately realistic models which include the free surface, bottom topography, stratification and currents. Many aspects of these important strips of the ocean are only likely to be understood via numerical models. C. N. K. Mooers* (University of Delaware) described a linearized two-layer model which can be adjusted to fit a wide range of conditions. A tanh profile was used for the depth and sech^2 profiles for the currents in the two layers. The adjustable parameters include the depths of the two layers, far from the shelf, the density jump, the current intensities and widths, the longshore wavenumber, the wave frequency, and the nature of the incident waves. By employing finite-difference approximations, the computational problem was reduced to solving large linear matrix equations. For incident barotropic waves numerical solutions and physical interpretations (particularly in terms of baroclinic variability) were presented for the dependence of the reflected and trapped modes upon each of the current shear, the angle of incidence, and the wave frequency.

L. A. Mysak gave a progress report on joint work with J. A. Helbig* (University of British Columbia) which dealt with low-frequency oscillations in the Strait of Georgia, British Columbia. In a recent study by Chang *et al.* (1976) it was found that nearly half of the horizontal kinetic energy in a set of current records for 18 months from one cross-section of the Strait of Georgia was contained in current oscillations possessing periods in the range 4–100 days. In a first attempt to explain these observations, a simple theory for topographic planetary waves in a two-layer variable-depth channel was presented. While the theory gave wave periods comparable to those observed, the predicted vertical distribution of the horizontal kinetic energy (a bottom-trapped motion) was the opposite of what has been observed. Various explanations for this discrepancy were offered, the most likely being the neglect of the observed mean flow in the model. Mysak stated that preliminary results from a baroclinic instability model for the Strait of Georgia suggest that the current oscillations may be associated with unstable waves on the mean flow.

7. Long-period barotropic waves

In the Australian setting, work on long-period barotropic waves has been dominated by continental-shelf waves (Hamon 1966; Buchwald & Adams 1968). These waves have a velocity of the order of 1 m/s and a period of a few days. Their existence was deduced and their properties were studied by examining tide gauge records from the east and west coasts of Australia. All of the work described in this section is Australian; some of it provides further insight into the theory of these shelf waves, but observations of other, unexplained long-period waves on the south coast are also included.

V. T. Buchwald* (University of New South Wales) discussed the scattering of a prescribed shelf mode incident upon a small coastal irregularity of compact support. On the basis of the Born approximation, he obtained a Fourier integral representation of the scattered field which gave two parts: a continuous spectrum (due to a pair of branch cuts) and a discrete spectrum (other shelf wave modes due to poles). In particular he noted that, if one of the scattered modes has zero group velocity at the incident wave frequency, then resonance occurs. Buchwald offered this mechanism (among others) as a possible reason for the observed sudden drop in coherence between the east Australian coast sea level and wind stress at periods of around four days (Hamon 1976).

N. G. Barton (University of New South Wales) gave a progress report on joint work with V. T. Buchwald* which dealt with the nonlinear generation of shelf waves. Barton showed that a time-dependent motion generated by a periodic wind blowing along an interior shelf (i.e. a long escarpment in the open ocean) and a first-mode free interior shelf wave could interact nonlinearly to produce a second-mode interior shelf wave. Then, provided the frequency of this forced wave was close to that for zero group velocity of this mode, resonance could occur. However, the precise nature of the growth at resonance had yet to be determined. Barton conjectured that the same sort of resonance could also occur on a coastal shelf, a mechanism which may provide another explanation of the observed coherence drop mentioned in the preceding paragraph.

In laboratory experiments, Caldwell & Eide (1976) observed long-period resonances around an island with a shelf in a rotating cylindrical container. V. T. Buchwald & W. K. Melville* (University of New South Wales) developed the theory of both short- and long-period oscillations (motions of edge, Kelvin and shelf wave type) in such a container by taking into account the horizontal divergence and the outer container wall. Buchwald showed that the resulting shallow-water equations for circularly travelling waves in the shelf and constant-depth regions were most simply solved by the method of Frobenius. By matching the surface elevation and radial transport at the shelf's edge, the eigenfrequency relation (resonance condition) was determined. Excellent agreement with experiment was obtained for the first few long-period modes. However the theory was found to break down for very small islands, probably because of frictional and nonlinear effects.

R. H. J. Grimshaw* (Melbourne University) presented a survey of some recent results for the effects of finite amplitude on shelf waves. First long shelf waves were discussed in the case where weak nonlinearities balance weak dispersion; then the waves are described by a Korteweg–de Vries equation. This equation is derived by holding the divergence parameter fixed while the limit $L/\lambda \to 0$ is taken in the full nonlinear shallow-water equations (L = shelf width and λ = wavelength). This distinguishes his approach from that of Smith (1972), who also allowed the divergence parameter to tend to zero with L/λ. Grimshaw next discussed the stability of shelf waves to modulation. A theory was developed in which the amplitude evolves on a time scale inversely proportional to the square of the wave amplitude, and is governed by a non-linear Schrödinger equation. He then demonstrated the existence of an instability due to resonance with the mean flow. Finally, in discussing wave-induced mean flows, Grimshaw showed that the governing equations for the mean flow are considerably simplified when formulated in terms of the Langrangian rather than the Eulerian mean flow.

W. D. McKee* (University of New South Wales) began his talk on continental shelf waves and currents by explaining, with commendable clarity, the importance of critical layers and discontinuous normal modes for inviscid parallel shear flows. For non-divergent barotropic flows the inclusion of rotation and of depth topography does not change the essential mathematical structure of the equations governing small disturbances. Indeed, the analogue of the Rayleigh criterion is simply that a necessary condition for instability is that there should be a zero of the gradient of the background potential vorticity (Niiler & Mysak 1971). In his talk McKee restricted attention to the stable case. Fourier–Laplace transforms were used to obtain solutions both for the initial-value problem and for the oceanographically more important forced problem. For a travelling wind stress pattern whose phase speed lies outside the range of the current speed, the contribution from the continuum of discontinous modes decays with time, leaving a forced response travelling with the same speed as the wind, together with oscillations from the discrete spectrum (shelf waves). However, if the phase speed of the forcing lies within the range of the current velocity then there is a large but finite response near the critical layers.

The continental-shelf waves on the east coast of Australia were also discussed by B. V. Hamon* (C.S.I.R.O., Cronulla) in one of the informal presentations. The model due to Gill & Schumann (1974) for the generation of shelf waves by the longshore component of wind stress was used to compute the shelf waves at Evans Head on the east coast of Australia. From records of the winds at six stations south of this port the mean sea level at Evans Head was calculated. These values were correlated with the adjusted mean sea level and the amplitude and phase compared. The results supported the hypothesis that the shelf waves on the east coast of Australia are generated by the longshore component of wind stress.

The final paper of the symposium was presented by the chairman of the International Symposium Committee, R. Radok (Horace Lamb Institute of Oceanography, Adelaide). Radok & G. Krause* have looked at the low frequency (subtidal) behaviour of daily mean sea levels and atmospheric pressures along the southern coast of Australia, Tasmania and Macquarie Island. They observed long-period waves with amplitudes as large as 1 m, periods of 5–8 days and lengths of 3500–5000 km travelling along the south coast of Australia, from Carnarvon on the Indian Ocean to Hobart in Tasmania, and then past Macquarie Island, between New Zealand and Antarctica. The travel time for these waves across the Great Australian Bight was observed to be about 2–3 days, which is somewhat shorter than the transit time for the weather systems to move across the same distance. Radok impressed upon us that these waves were not confined to the shelf region, but represented some open-ocean phenomenon of very large scale. Also he pointed out that such motions may be masked by a spectral analysis of barometrically adjusted sea levels alone, and that the response of the ocean to atmospheric pressure distributions needs further study.

All the participants in this symposium are indebted to the Secretary-General of IUTAM, Professor Niordson, to the Chairman of the Scientific Committee, R. Radok, and to the other members of the Committee (C. Greated, Edinburgh; I. G. Jonsson, Copenhagen; M. S. Longuet-Higgins, Cambridge; J. W. Miles, La Jolla; and O. S. Vasiliev, Novosibirsk) for promoting, organizing and ensuring the smooth running of this valuable meeting.

Appendix. Papers presented at the symposium

Alexeev, A. S. & Gusiakov, V. K. Numerical simulation of tsunami generation and propagation in an ocean with a real bathymetry.

Barton, N. G. & Buchwald, V. T. The nonlinear generation of shelf waves.

Bowen, A. J. Wave–wave interactions near the shore.

Buchwald, V. T. The diffraction of shelf waves by an irregular coastline.

Buchwald, V. T. & Melville, W. K. Resonance of shelf waves near islands.

Christiansen, P. L. Diffraction of gravity waves by ray methods.

Chwang, A. T. & Wu, T. Y. Cylindrical solitary waves.

Fenton, J. D. & Mills, D. A. Shoaling waves: numerical solutions of exact equations.

Grimshaw, R. H. J. Nonlinear aspects of shelf waves and mean currents.

Hamon, B. V. Generation of shelf waves on the east Australian coast by wind stress.

Helbig, J. A. & Mysak, L. A. Along the strait in thirty-six days.

Hibberd, S. & Peregrine, D. H. Surf and run-up.

Hornung, H. G. & Killen, P. Laboratory study of a stationary oblique plunging breaker for surfboard testing.

Jonsson, I. G. The dynamics of waves on currents over a weakly varying bed.

Kajiura, K. Local behaviour of tsunamis.

Krause, G. & Radok, R. Long waves on the Southern Ocean.

Longuet-Higgins, M. S. On breaking waves.

McKee, W. D. Continental-shelf waves in the presence of a sheared geostrophic current.

Mooers, C. N. K. The interaction of surface and internal tides with boundary currents.

Nagata, Y. Survey of theoretical research on tsunamis and observations of actual tsunamis in Japan.

Peregrine, D. H. & Thomas, G. P. Finite-amplitude waves on non-uniform currents.

Preisendorfer, R. W. Linear transport methods for long surface waves in canals, bays and oceans.

Provis, D. G. Experimental studies of wave refraction.

Roseau, M. Ondes internes évanescentes à l'infini dans un milieu liquide stratifié en rotation.

Smith, R. Triple roots and cusped caustics for surface gravity waves.

Tuck, E. O. Some classical water-wave problems in varying depth.

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